Stretching and Wiggling Liquids

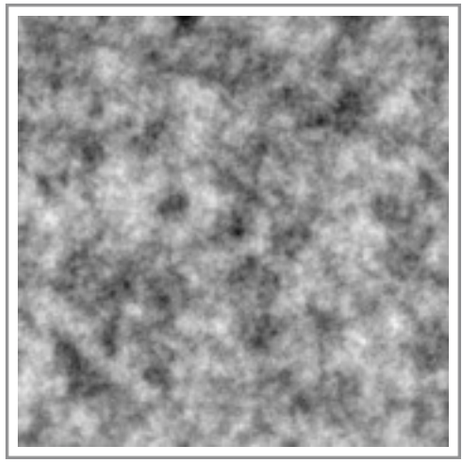
Doyub Kim*, Oh-young Song†, and Hyeong-Seok Ko*
*Seoul National University, †Sejong University

Presented by Doyub Kim
SIGGRAPH Asia 2009

http://www.trenterprises.com/company/wallpaper.html

Generating Complexity of Fluid

$\nabla \times$

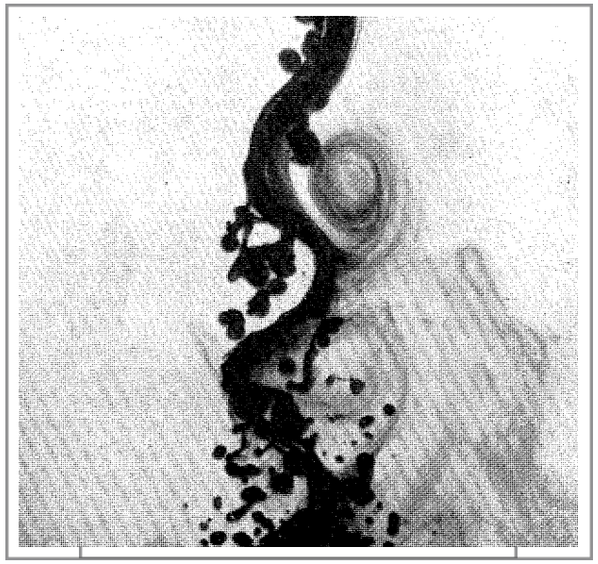


$+$

$=$

Selle et al. 2005
Kim et al. 2008

Generating Complexity of ***Liquid***



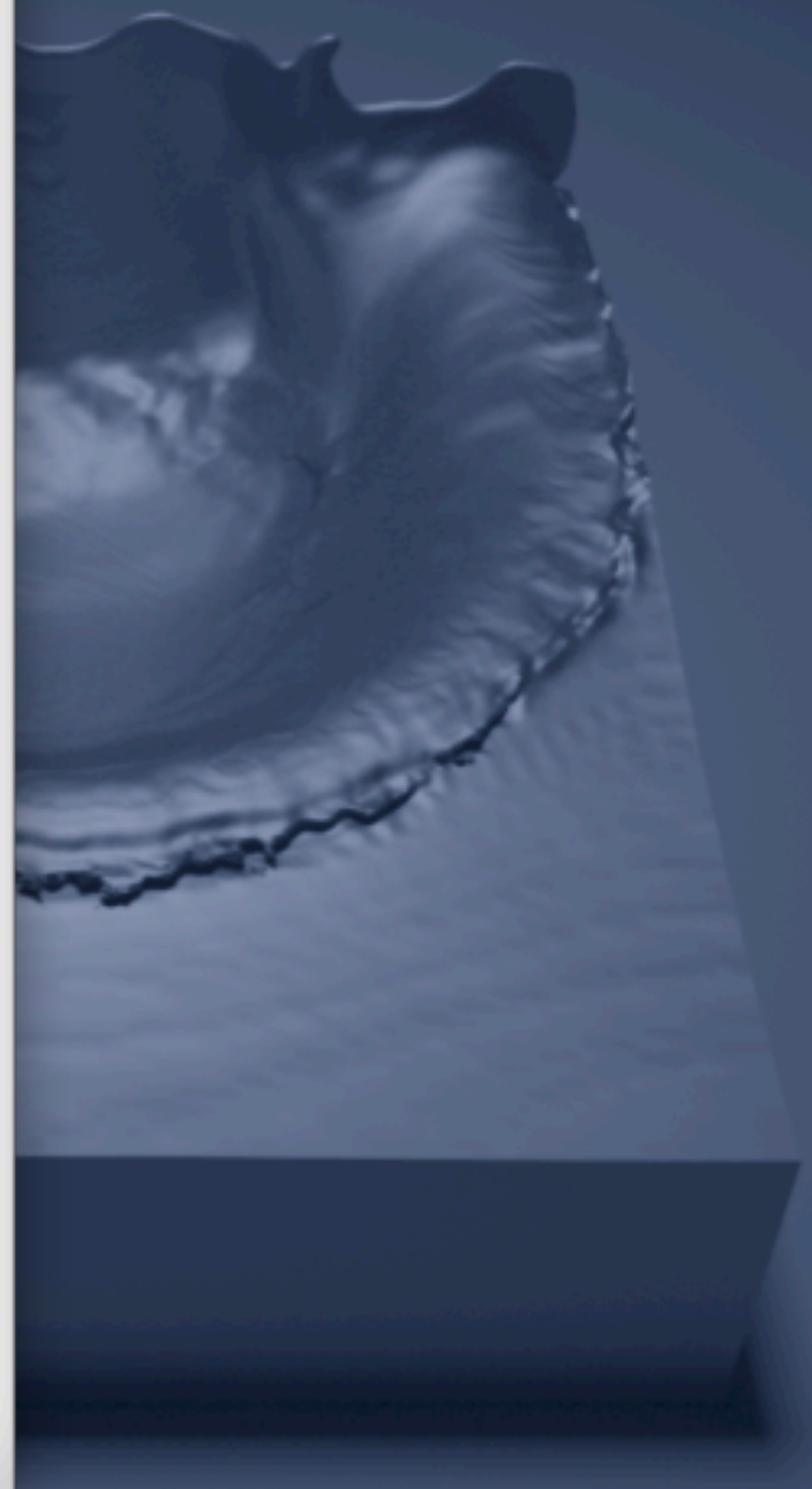
Vortex Sheet Method

+

=

Basic Fluid Simulation

- Level Set Method
- Stable Fluids Framework

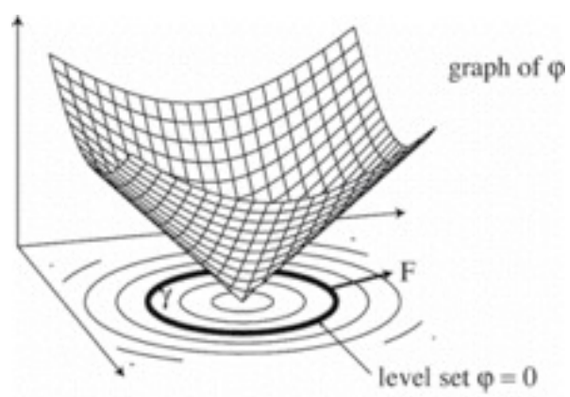


Level Set method

- Level set equations [Osher and Fedkiw 2001]

$$\frac{\partial \phi}{\partial t} + \mathbf{u} \cdot \nabla \phi = 0$$

$$|\nabla \phi| = 1$$

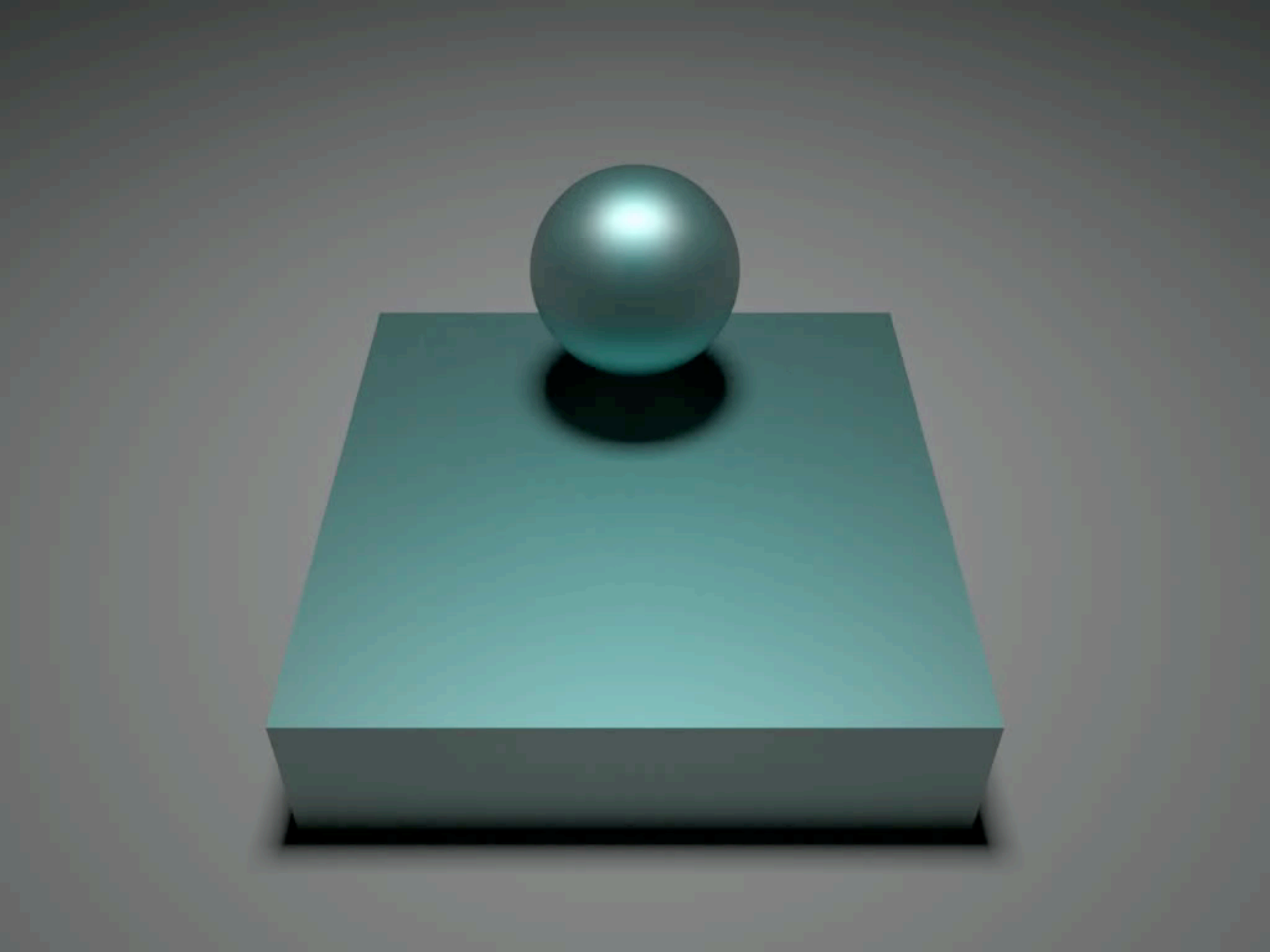


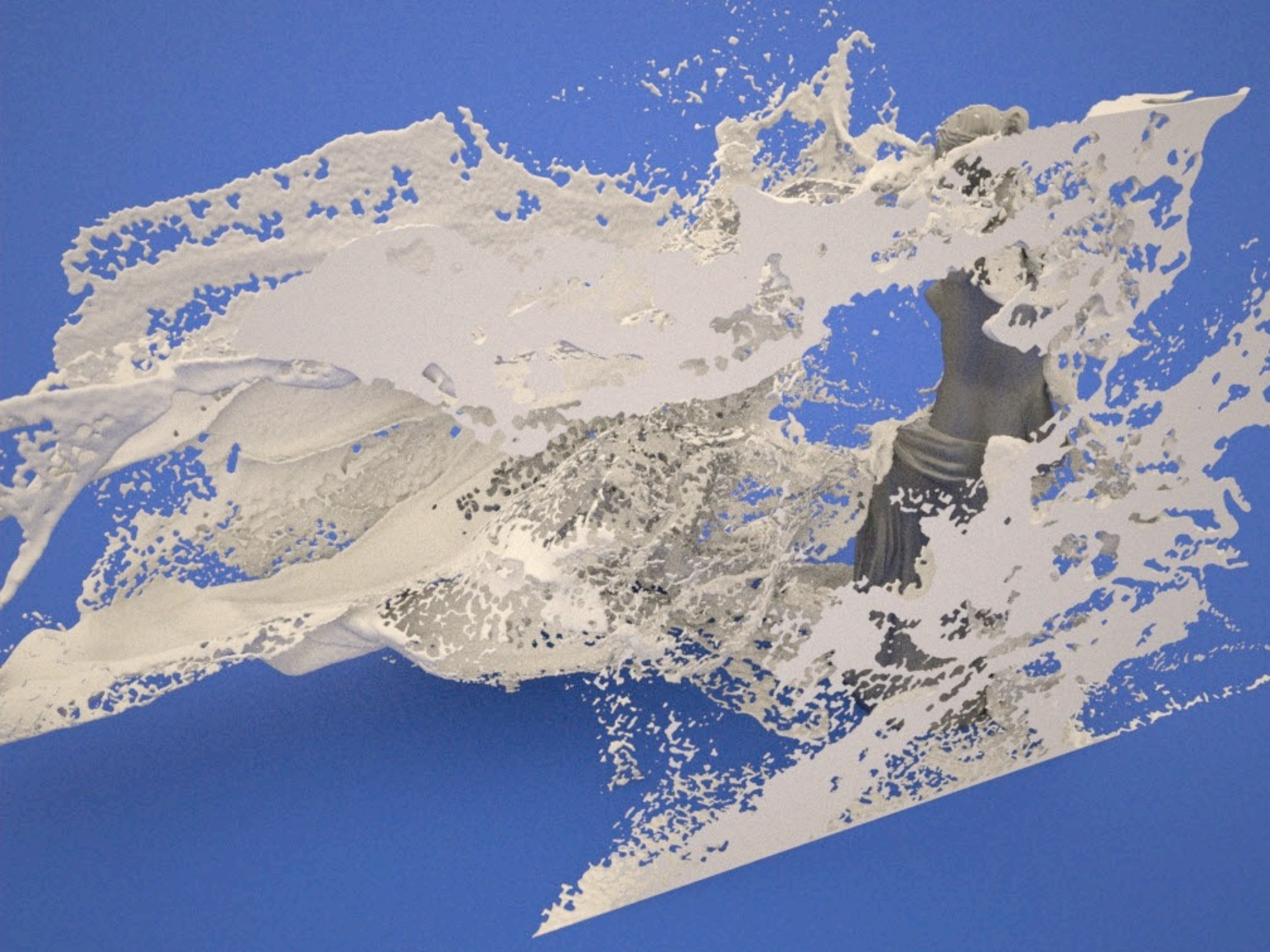
Stable Fluids

- Invicid Incompressible Navier-Stokes Equations

$$\mathbf{u}_t + (\mathbf{u} \cdot \nabla)\mathbf{u} + \nabla p = \mathbf{f}$$

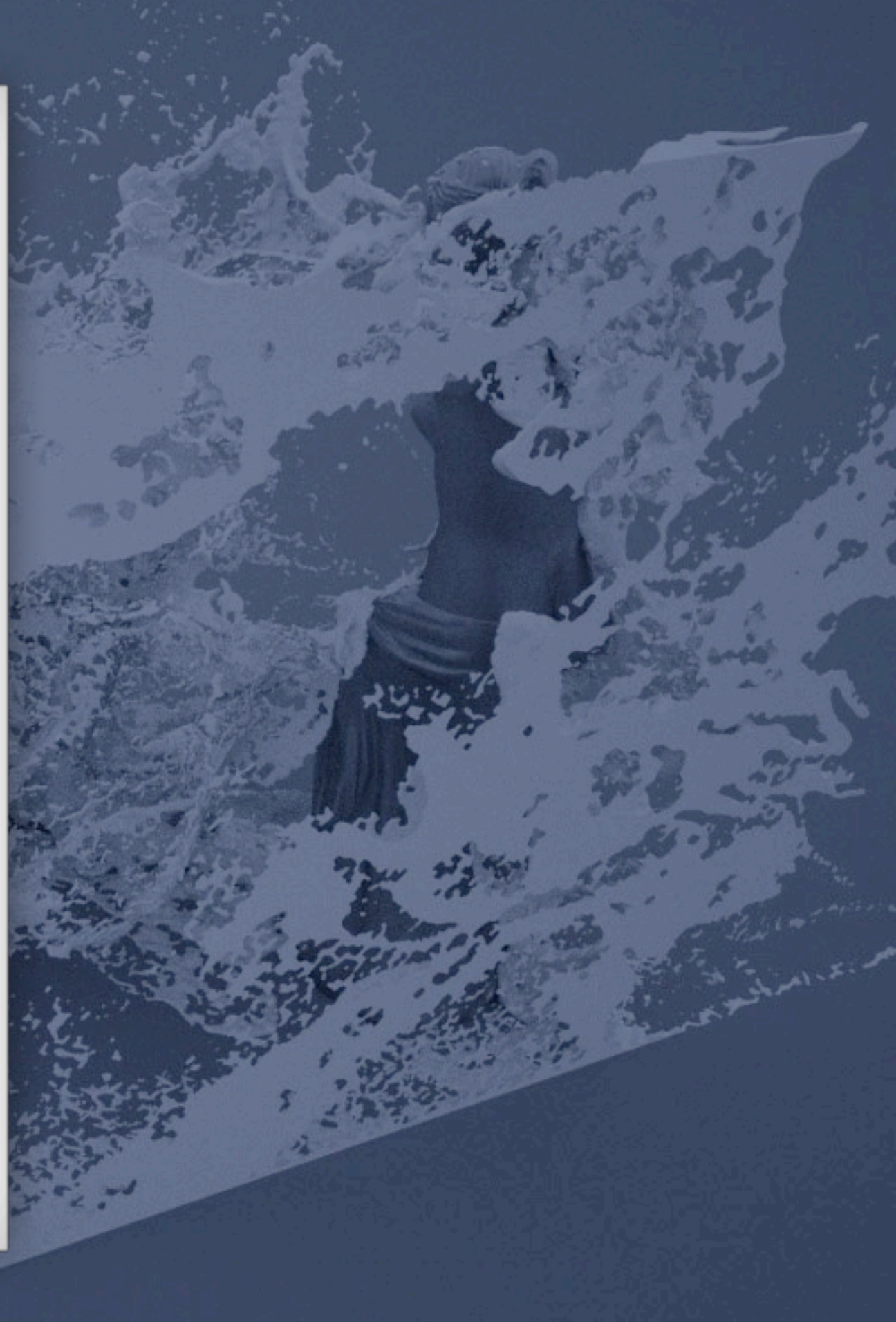
$$\nabla \cdot \mathbf{u} = 0$$





Generating and Capturing Complexity of Liquid

- Controllable Eulerian Vortex Sheet Method
- Super sampling with Liquid-biased Filtering



Generating Complexity

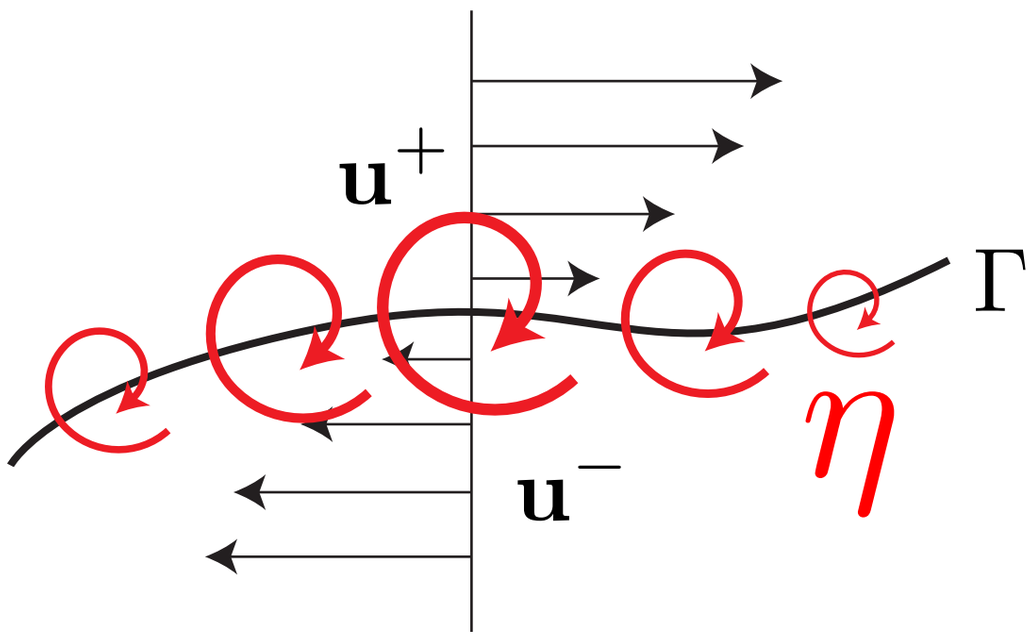
- Controllable Eulerian Vortex Sheet Method

Vortex

A vortex is a spinning, often turbulent, flow of fluid.

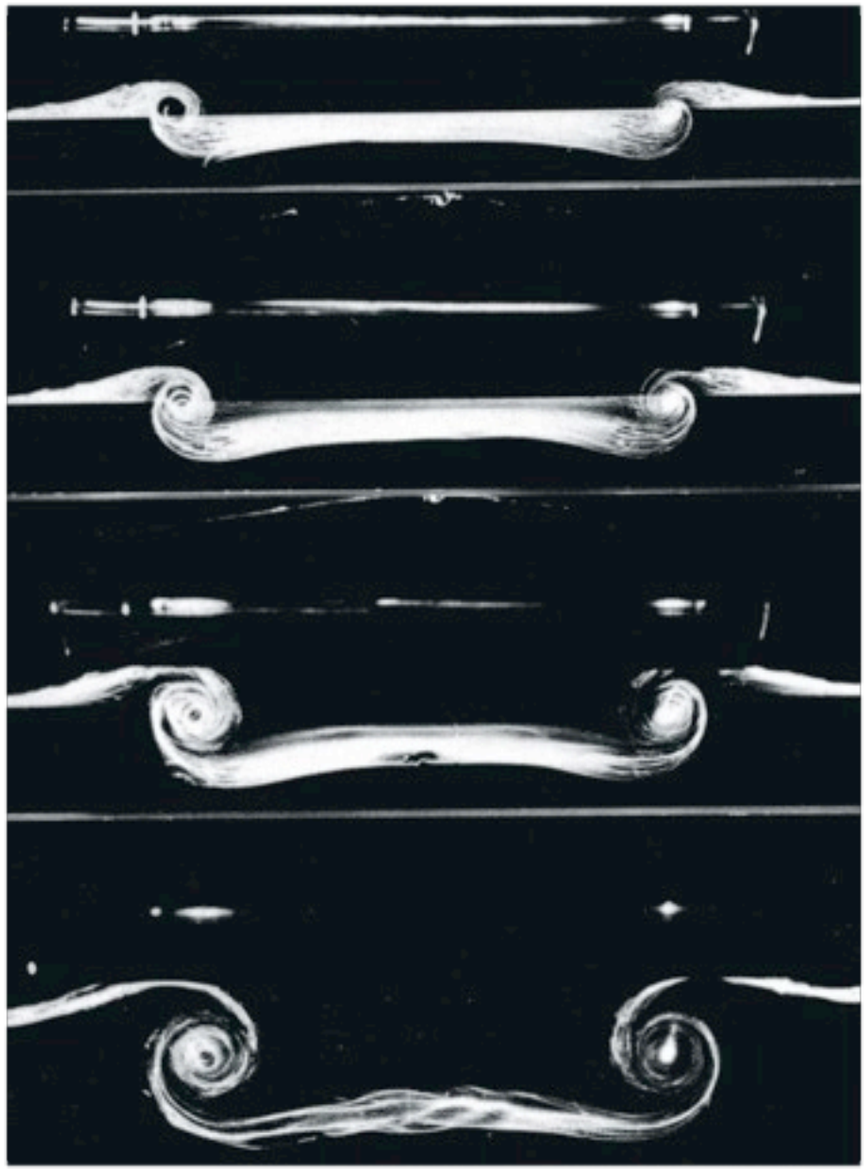
$$\omega = \nabla \times u$$

Vortex Sheet



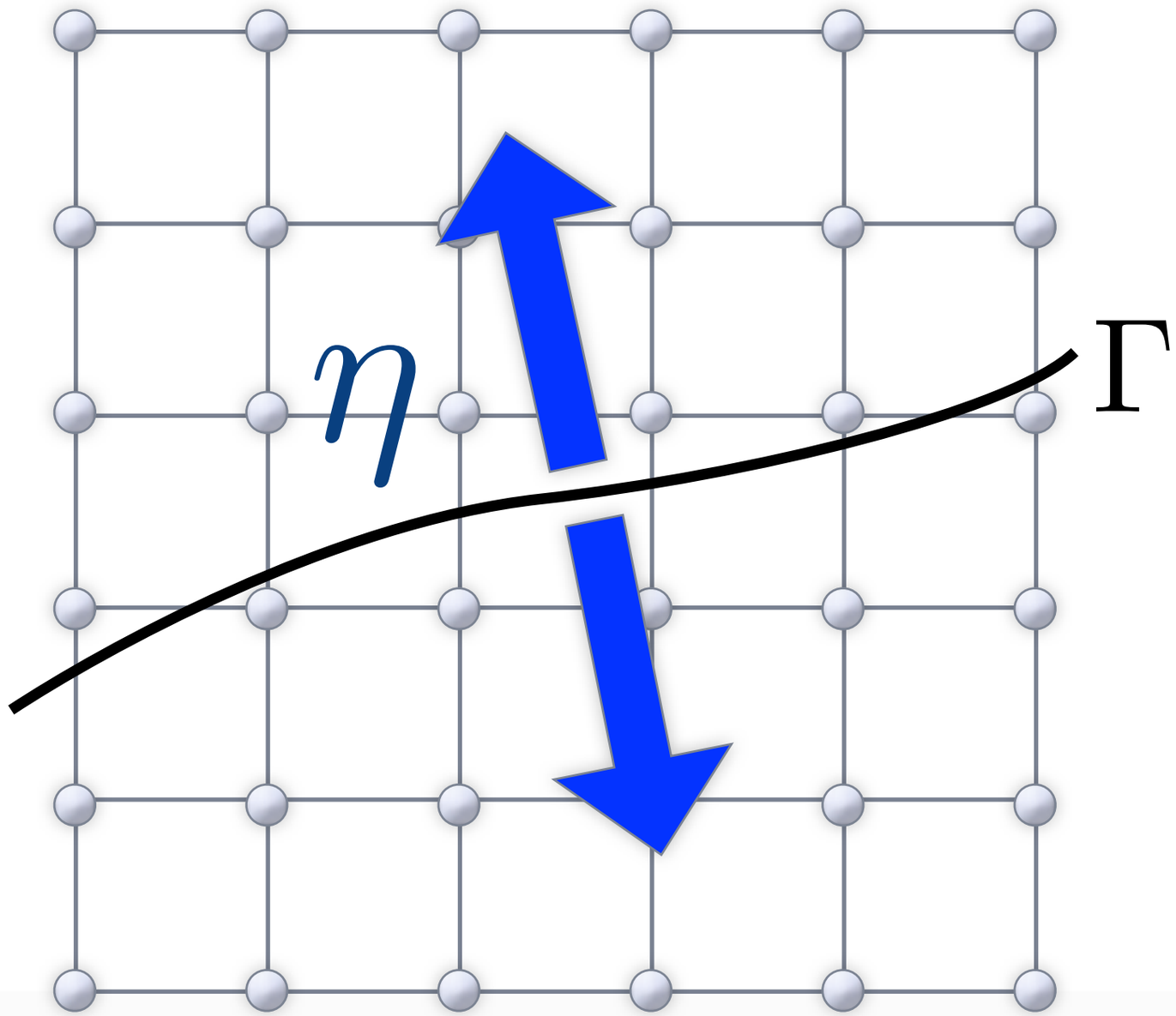
$$\eta = \mathbf{n} \times (\mathbf{u}^+ - \mathbf{u}^-)|_\Gamma$$

Vortex Sheet Strength

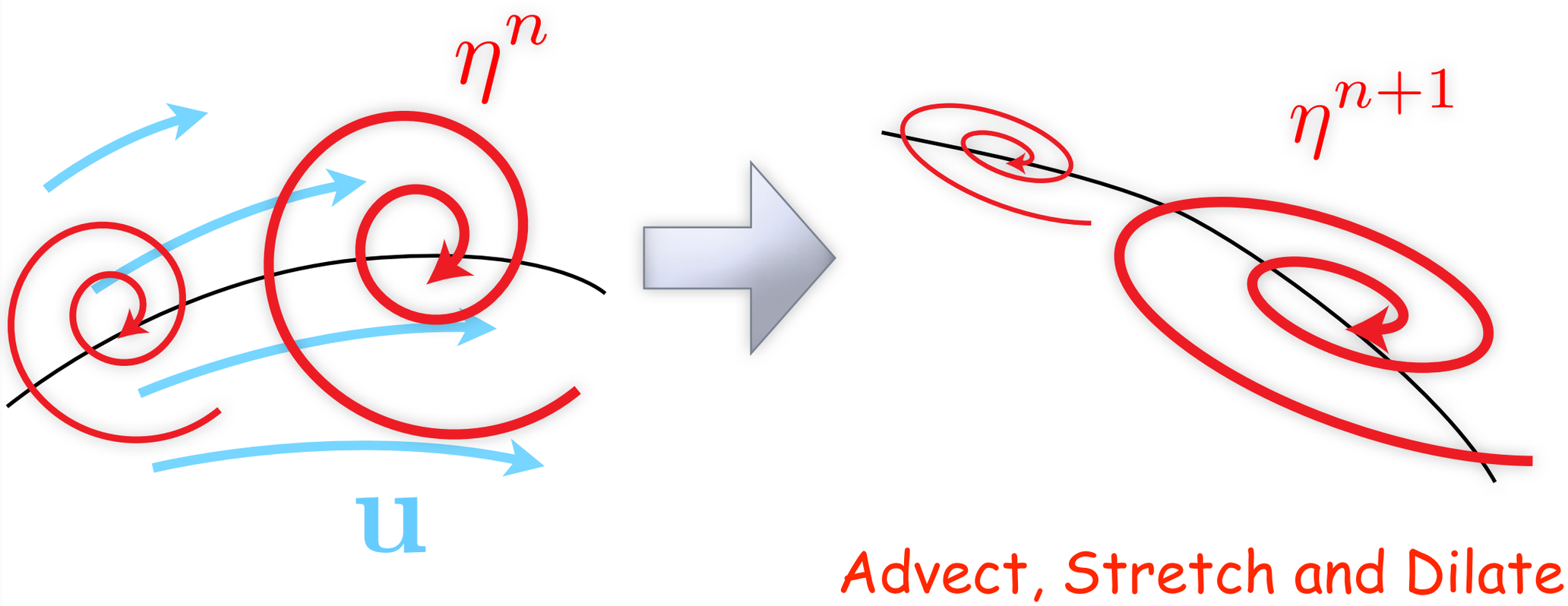


http://aeroacoustics.kaist.ac.kr

Eulerian Vortex Sheet



Evolution of Vortex Sheet



Eulerian Vortex Sheet Method

- Evolution of vortex sheet [Herrmann 2003]

$$\eta_t + \mathbf{u} \cdot \nabla \eta = -\mathbf{n} \times \{(\eta \times \mathbf{n}) \cdot \nabla \mathbf{u}\} + \mathbf{n}\{(\nabla \mathbf{u} \cdot n) \cdot \eta\} + \mathbf{S}$$

Advection

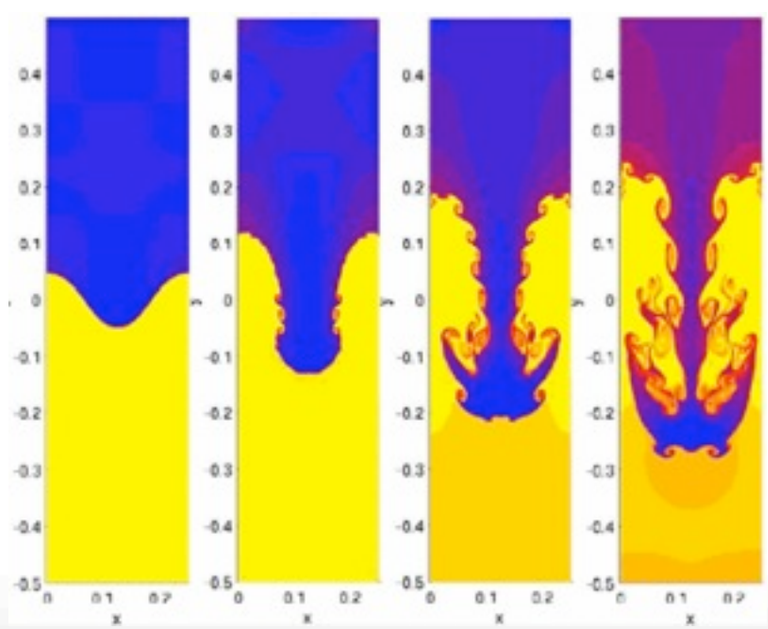
Stretch and Dilatation

Baroclinity Effect

- Baroclinic vector

$$\nabla p \times \nabla \rho$$

  - A source term which appears in the vorticity equation whenever iso-density surface and iso-pressure surface are not aligned.



Eulerian Vortex Sheet Method

- Evolution of vortex sheet

$$\eta_t + \mathbf{u} \cdot \nabla \eta = -\mathbf{n} \times \{(\eta \times \mathbf{n}) \cdot \nabla \mathbf{u}\} + \mathbf{n}\{(\nabla \mathbf{u} \cdot n) \cdot \eta\} + \mathbf{S}$$

Advection

Stretch and Dilatation

where,

$$\mathbf{S} = \cancel{2}\cancel{A}\mathbf{n} \times (\mathbf{a} - \mathbf{g})$$

Baroclinity

Applying Vortex Sheet Strength

- Vortex sheet strength *to* vorticity

$$\boxed{\omega(\mathbf{x})} = \int_V \boxed{\eta(\mathbf{x}')}\delta(\mathbf{x}-\mathbf{x}')\delta(\phi(\mathbf{x}'))|\nabla\phi(\mathbf{x}')|d\mathbf{x}'$$

Vortex Vortex Sheet Strength

- Vorticity *to* velocity
  - Similar to the vortex particle method [Selle et al. 2005]

$$\mathbf{f}_v(\mathbf{x}) = \alpha h(\mathbf{N} \times \omega)$$

Applying Vortex Sheet Strength

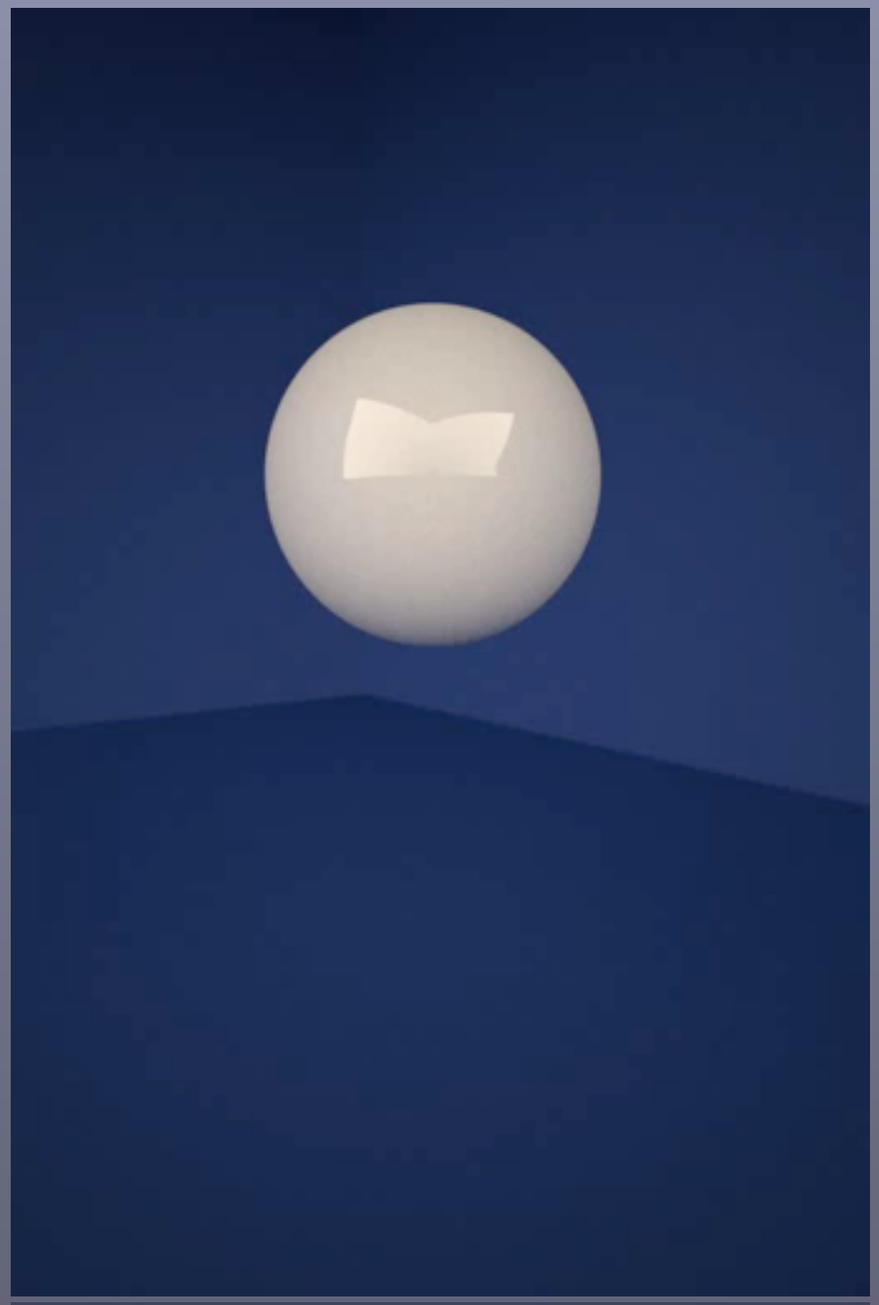
- Vortex sheet strength *to* vorticity

$$\boxed{\omega(\mathbf{x})} = \int_V \boxed{\eta(\mathbf{x}')}\delta(\mathbf{x}-\mathbf{x}')\delta(\phi(\mathbf{x}'))|\nabla\phi(\mathbf{x}')|d\mathbf{x}'$$

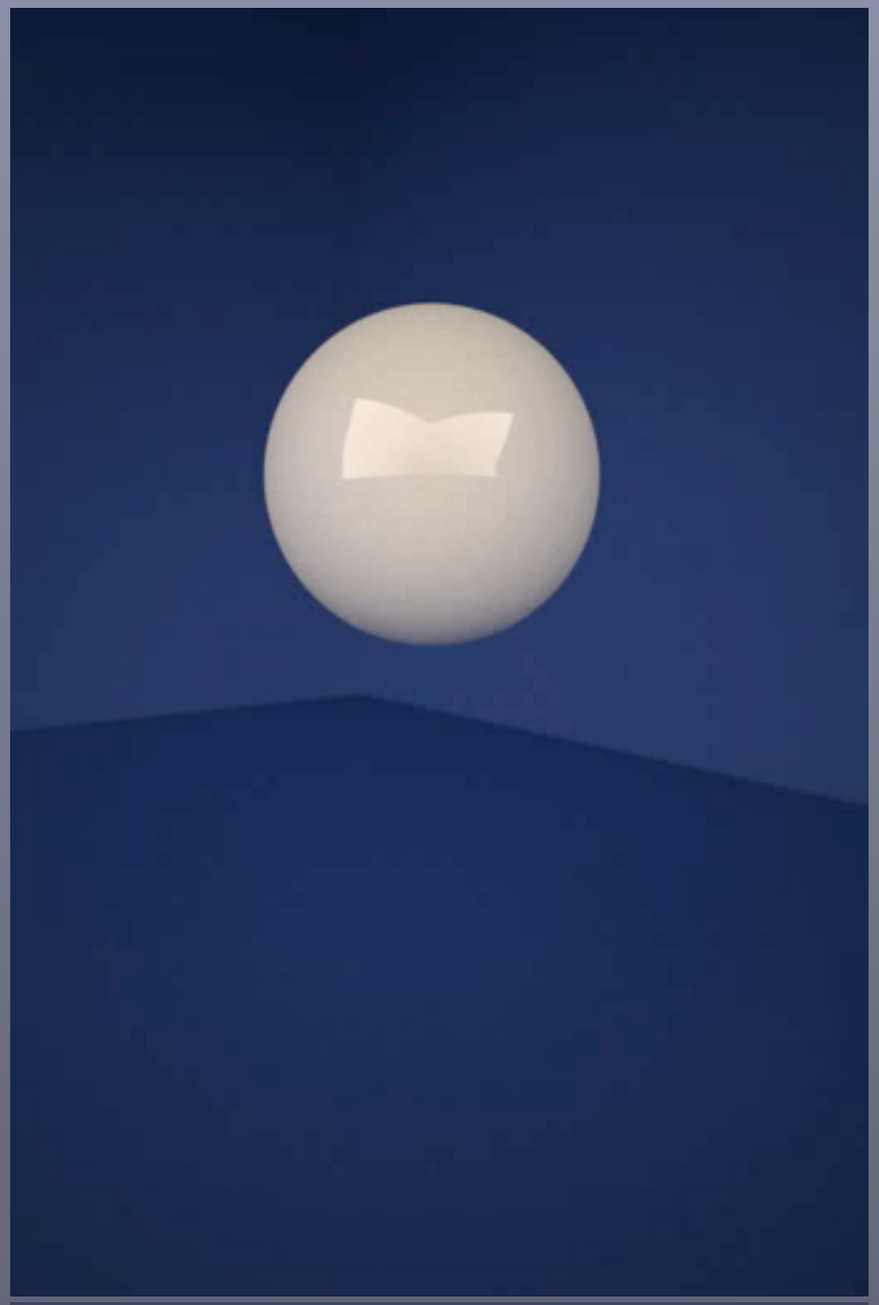
Vortex Vortex Sheet Strength

- Vorticity *to* velocity
  - Similar to the vortex particle method [Selle et al. 2005]

$$\mathbf{f}_v(\mathbf{x}) = \alpha h(\mathbf{N} \times \omega)$$



No Baroclinity



With Baroclinity

Turbulence Effect

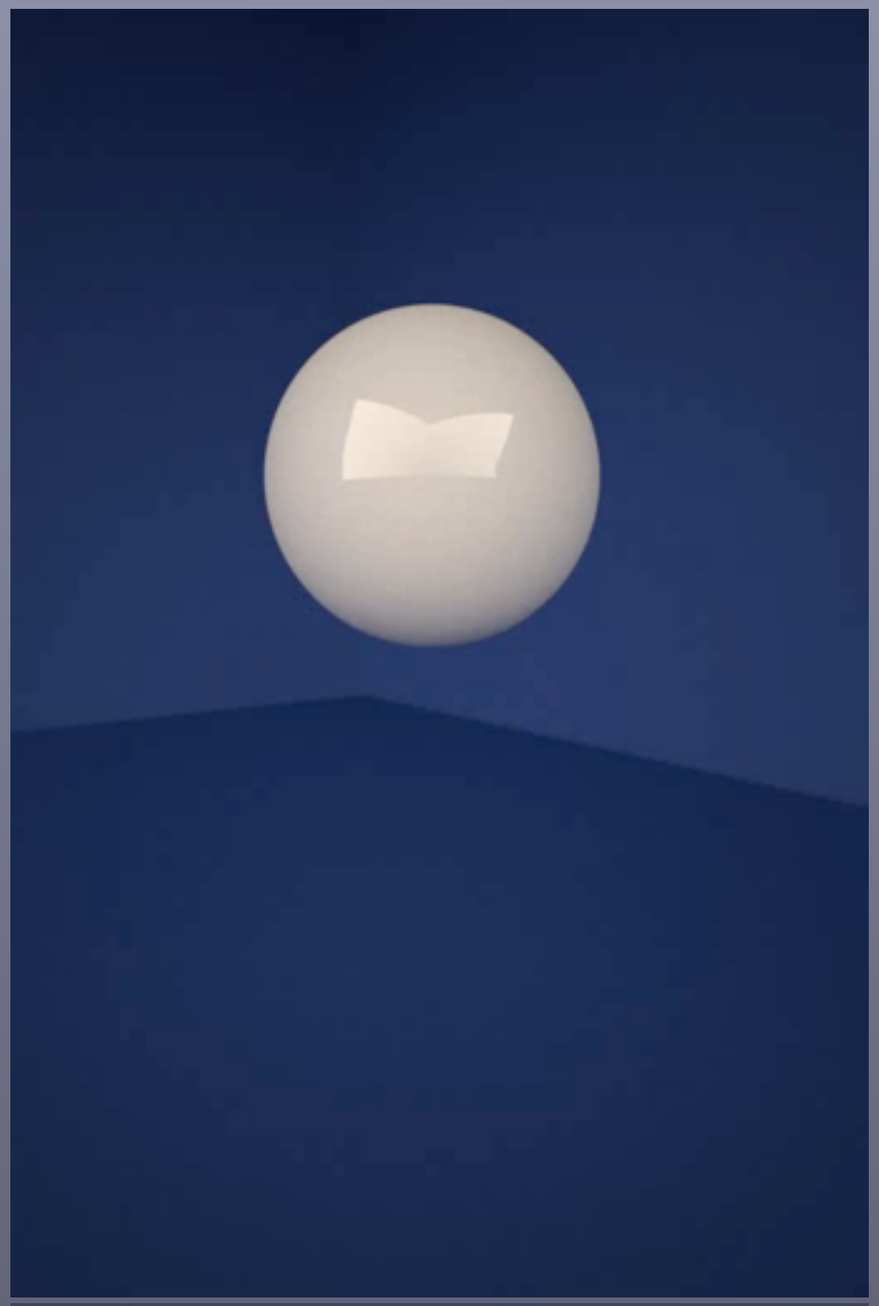
- Ambient Noise
  - Based on 4-D (time and space) Perlin noise

Noise Field

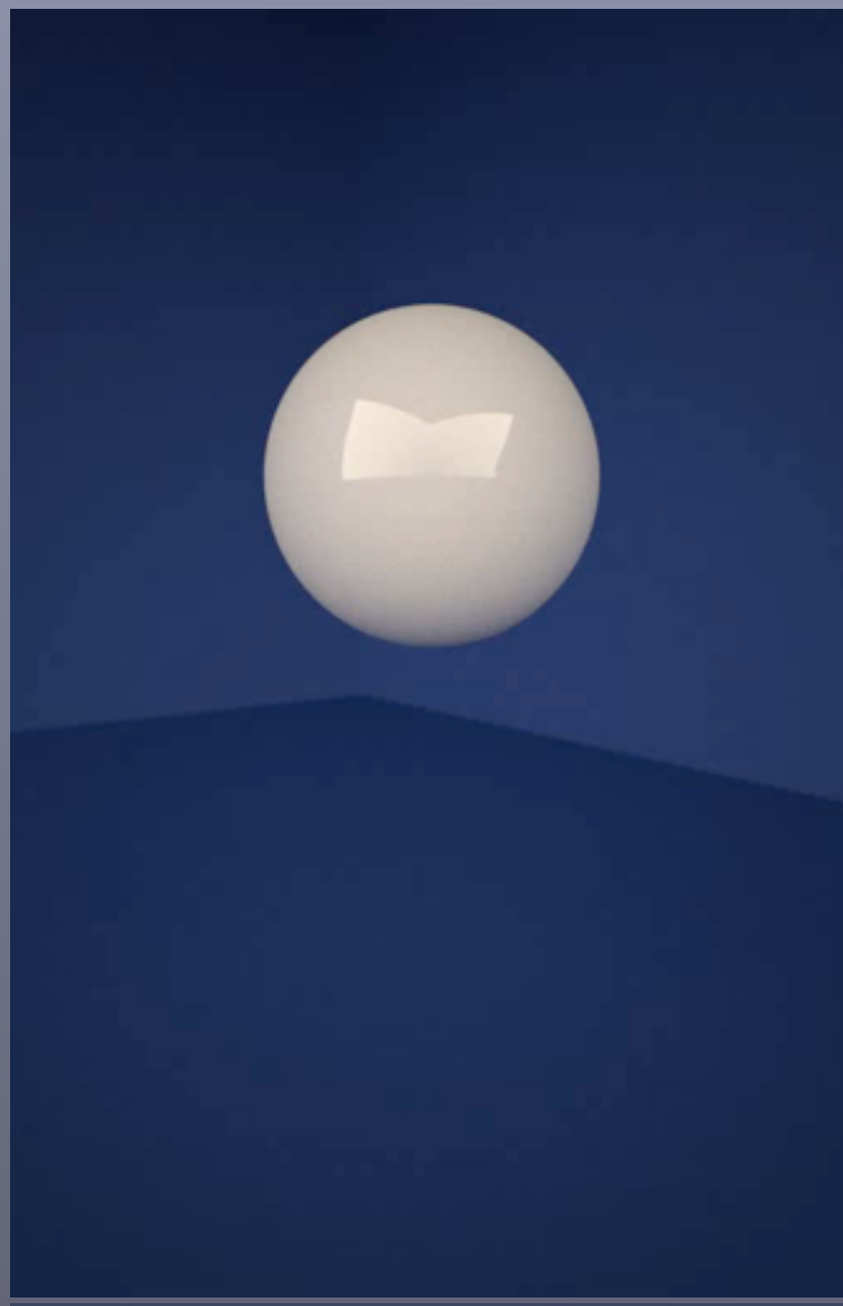
$$\hat{\eta} = \eta(1 + \mathbf{T}\mathbf{\Phi})$$

Projection

No Baroclinity



With Baroclinity



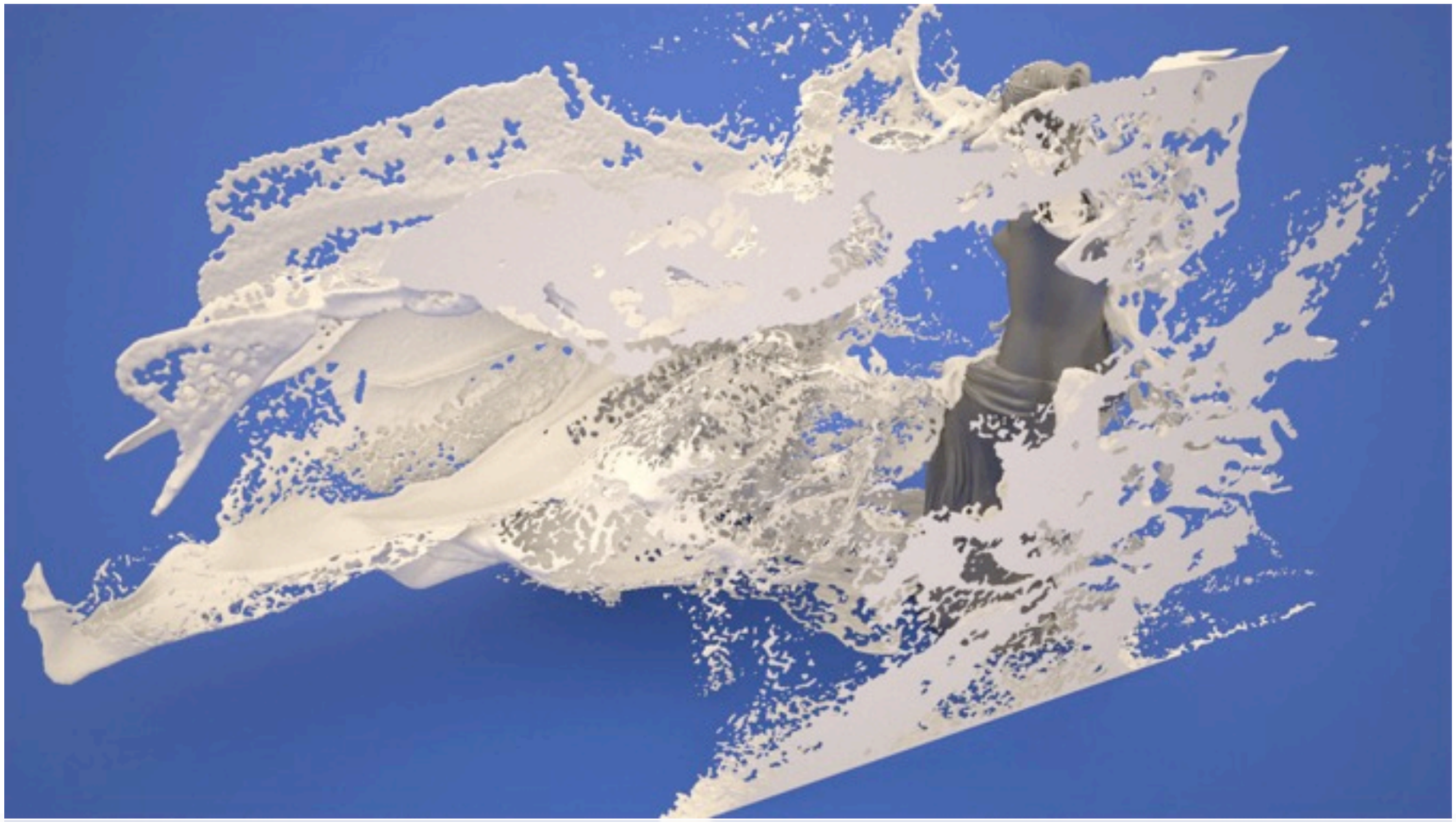
Baroclinity + Noise

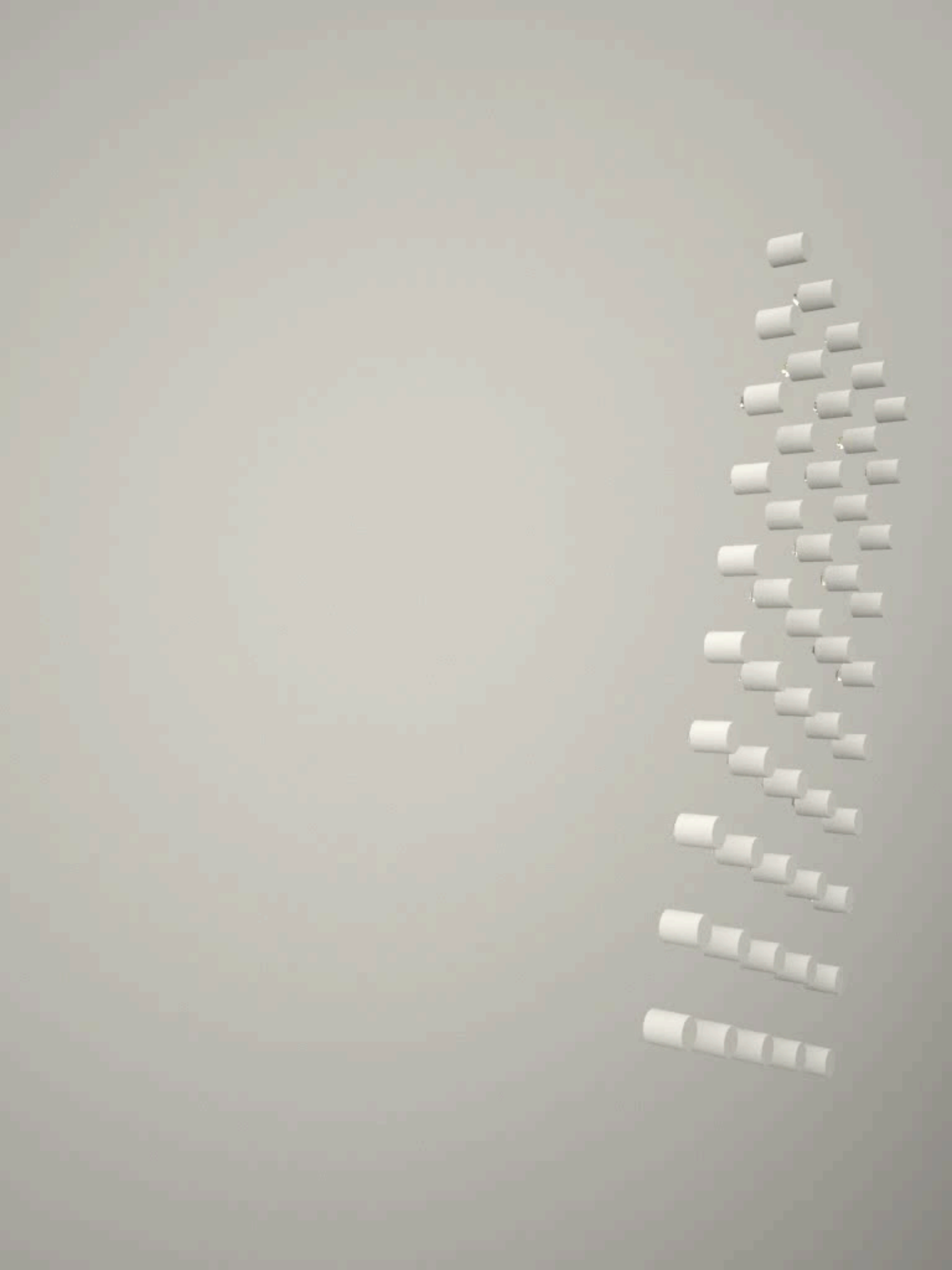
Capturing Complexity

- Super-sampling Approach
- Liquid-biased Filtering

Super-sampling Approach

- Use high-resolution grid for surface tracking

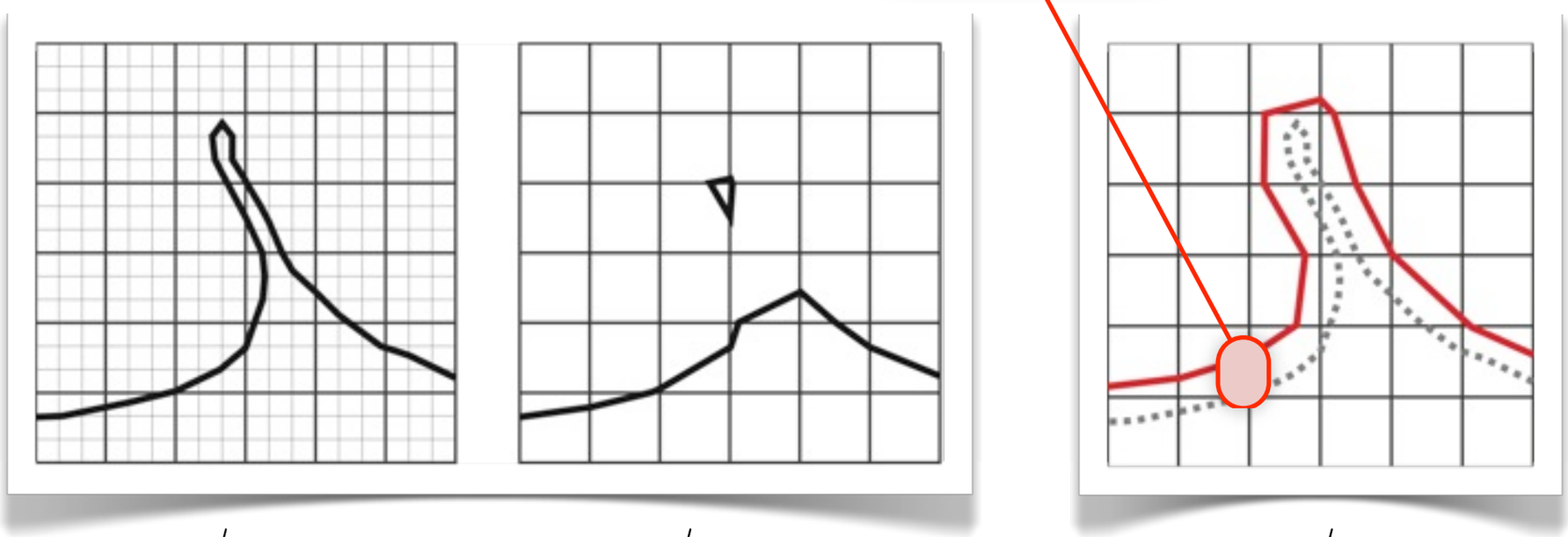




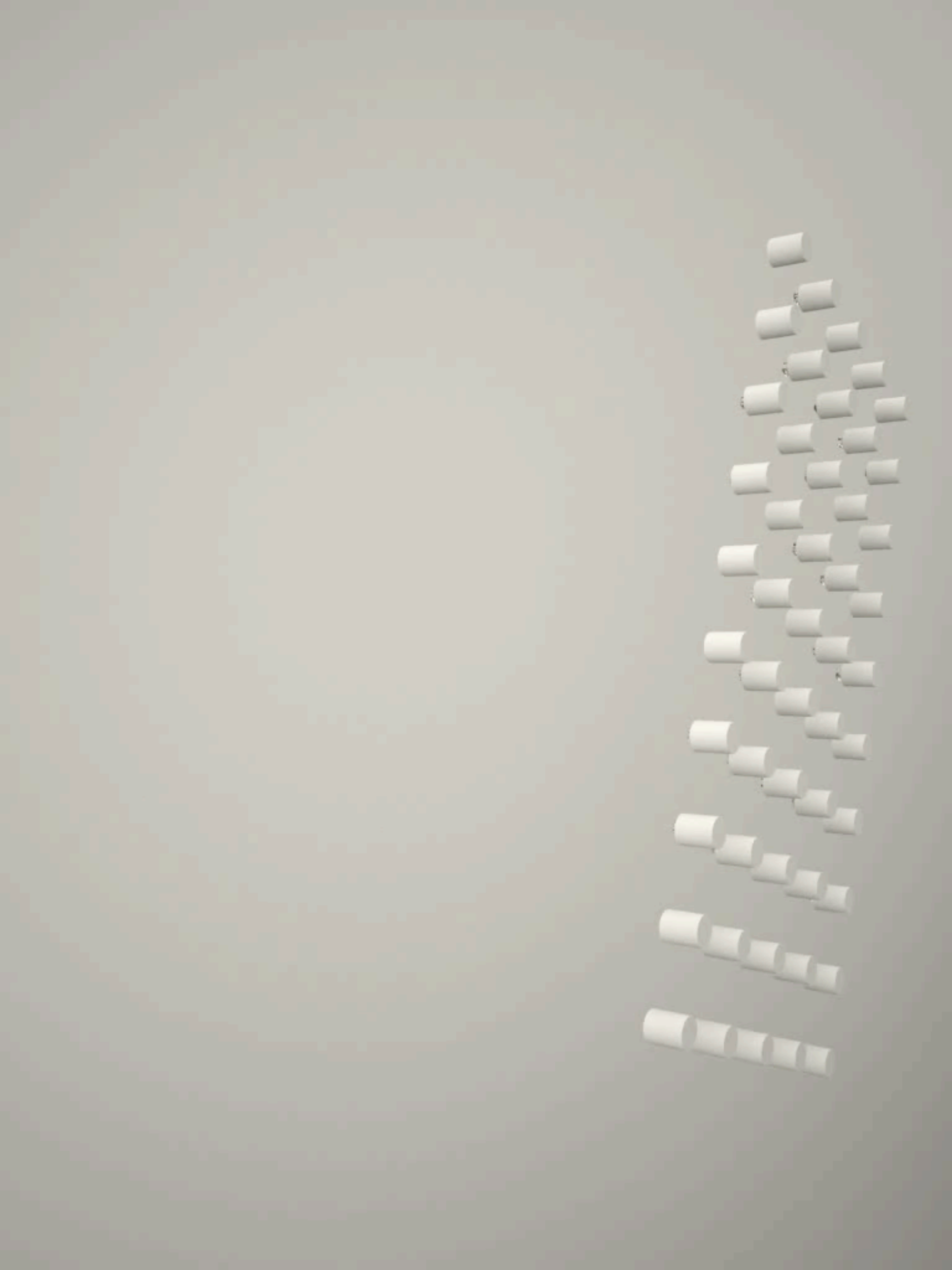
Liquid-biased Filtering

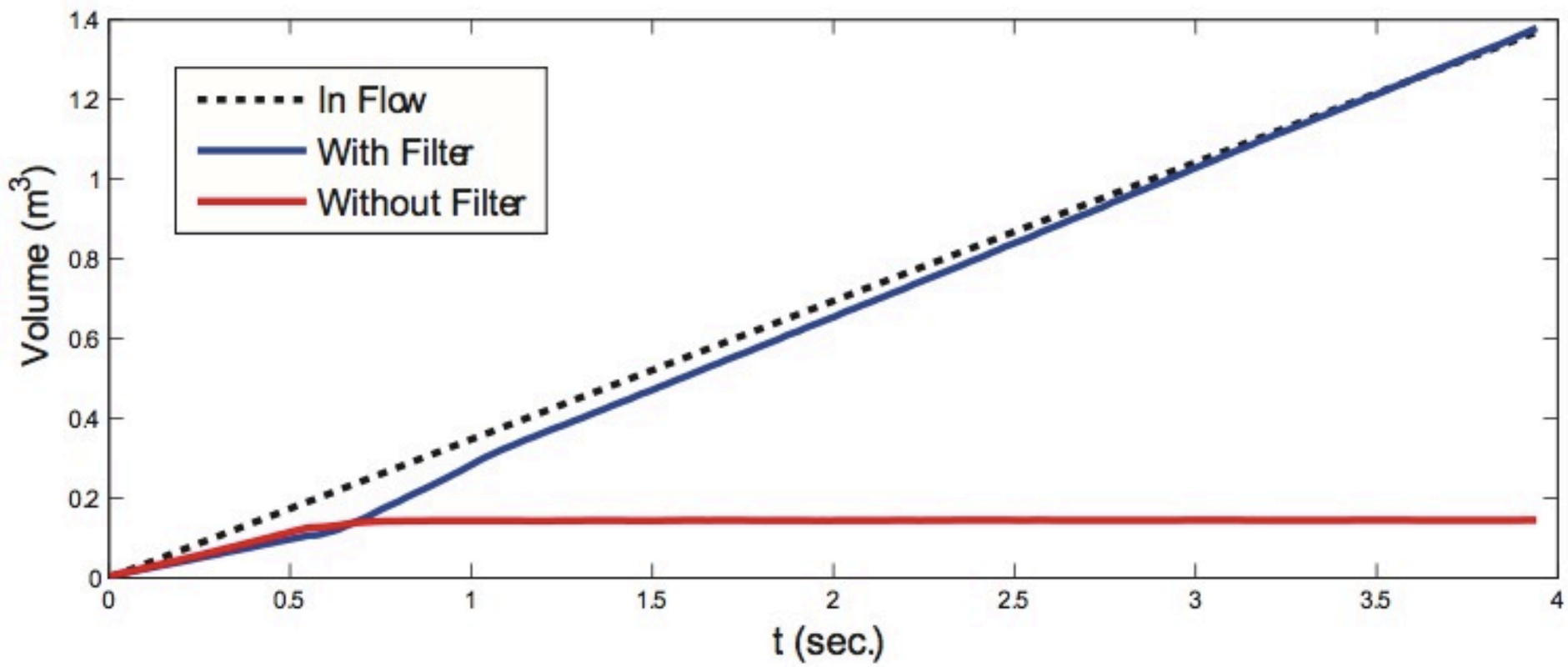
- To make filtered surface fully contains the original surface, we shift the zero level set, and then apply linear filter.

$$\phi_C = [\phi_D] \downarrow \boxed{-0.5h_C}$$



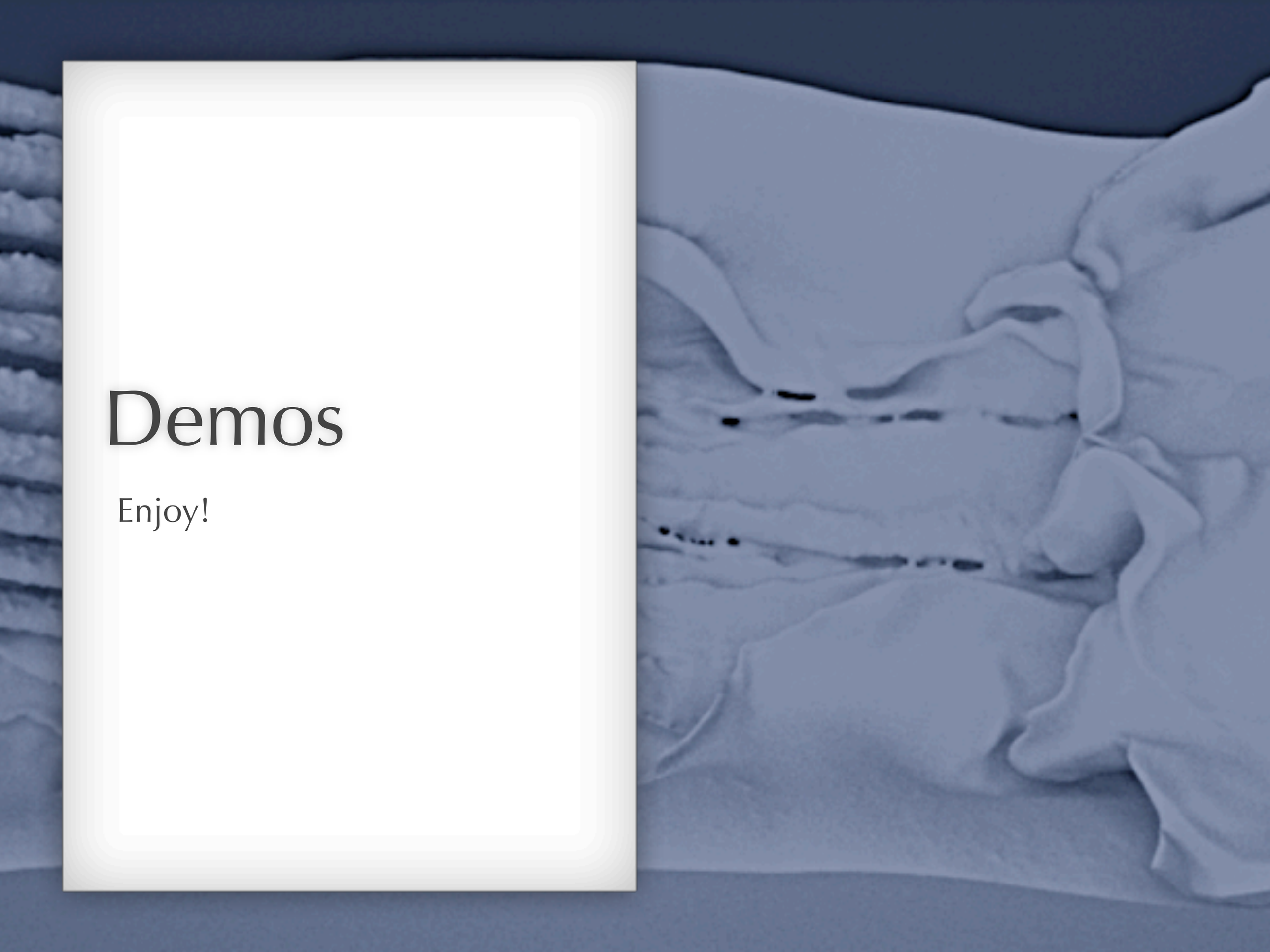
ϕ_D ϕ_C ϕ_C

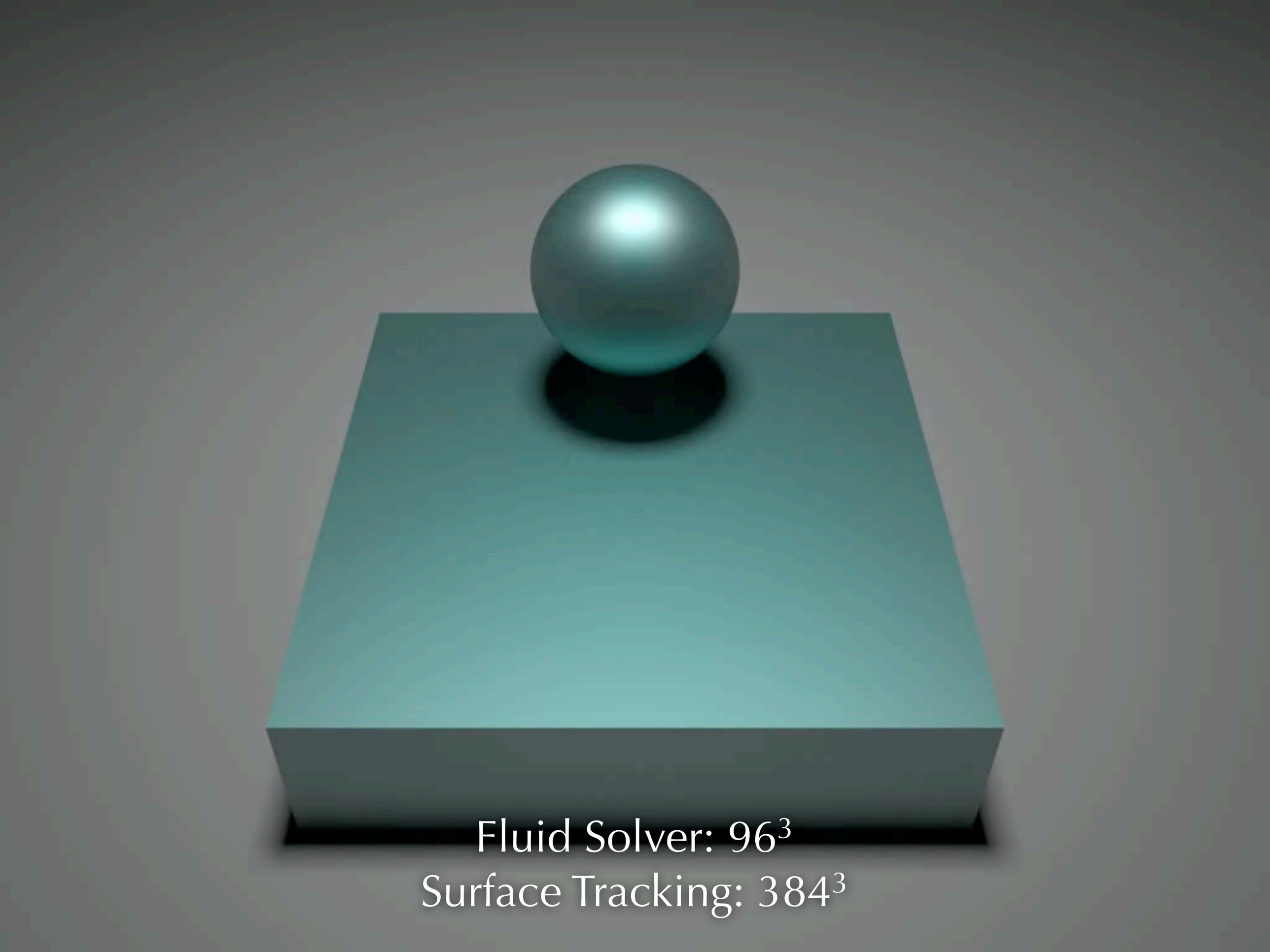


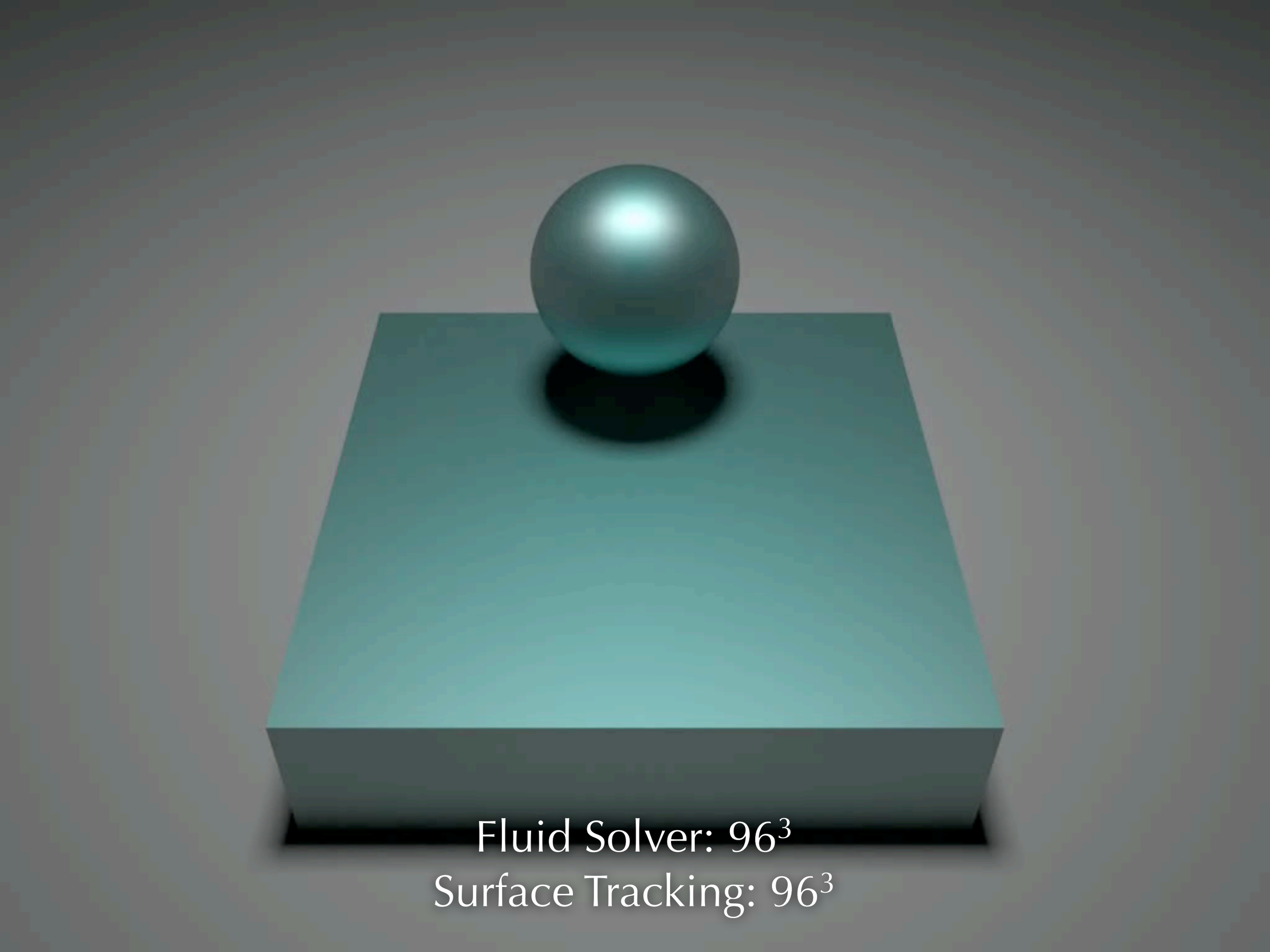

In Flow
With Filter
Without Filter
Volume (m^3)
t (sec.)
0
0.2
0.4
0.6
0.8
1
1.2
1.4
0
0.5
1
1.5
2
2.5
3
3.5
4

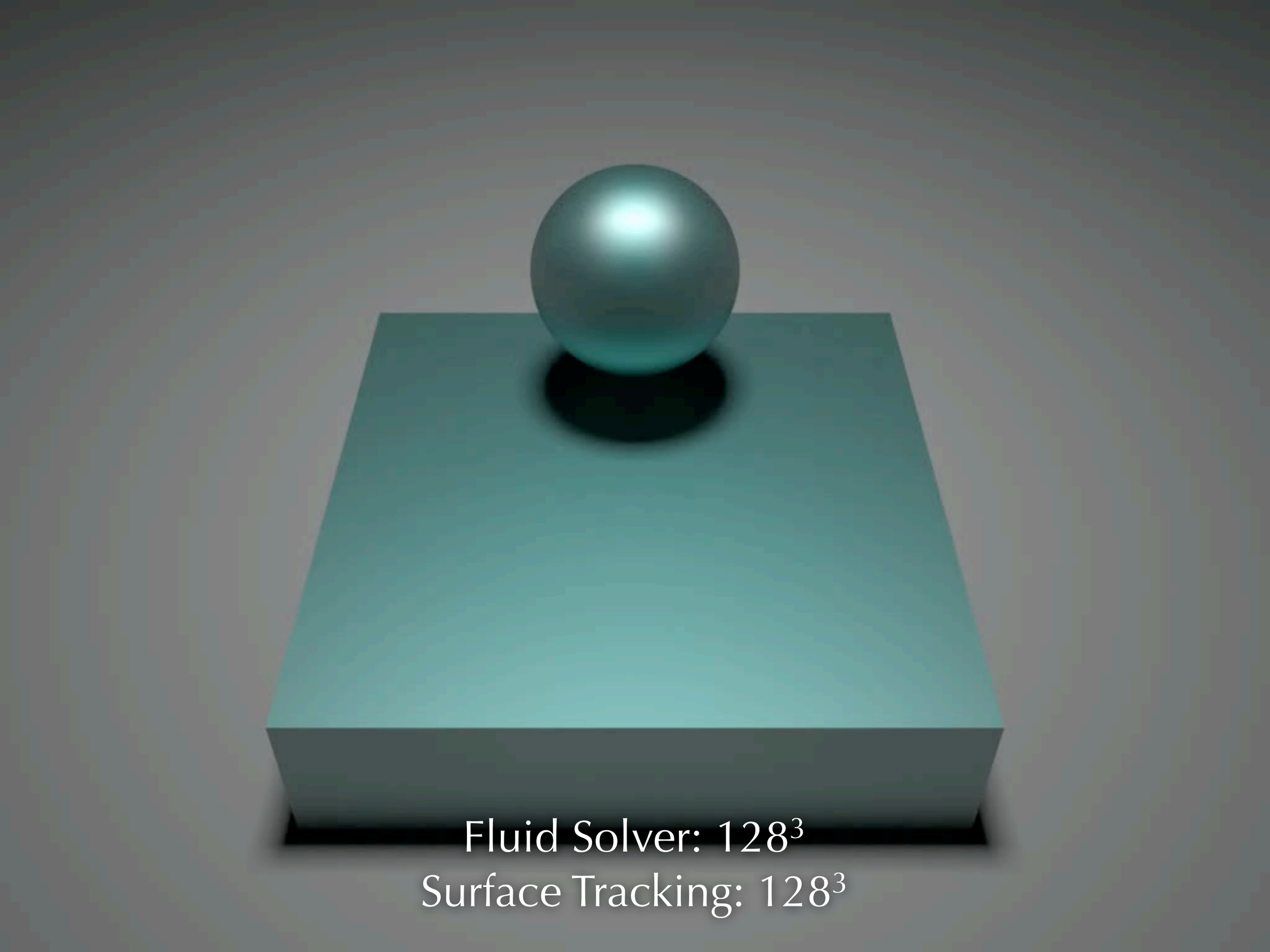
Demos

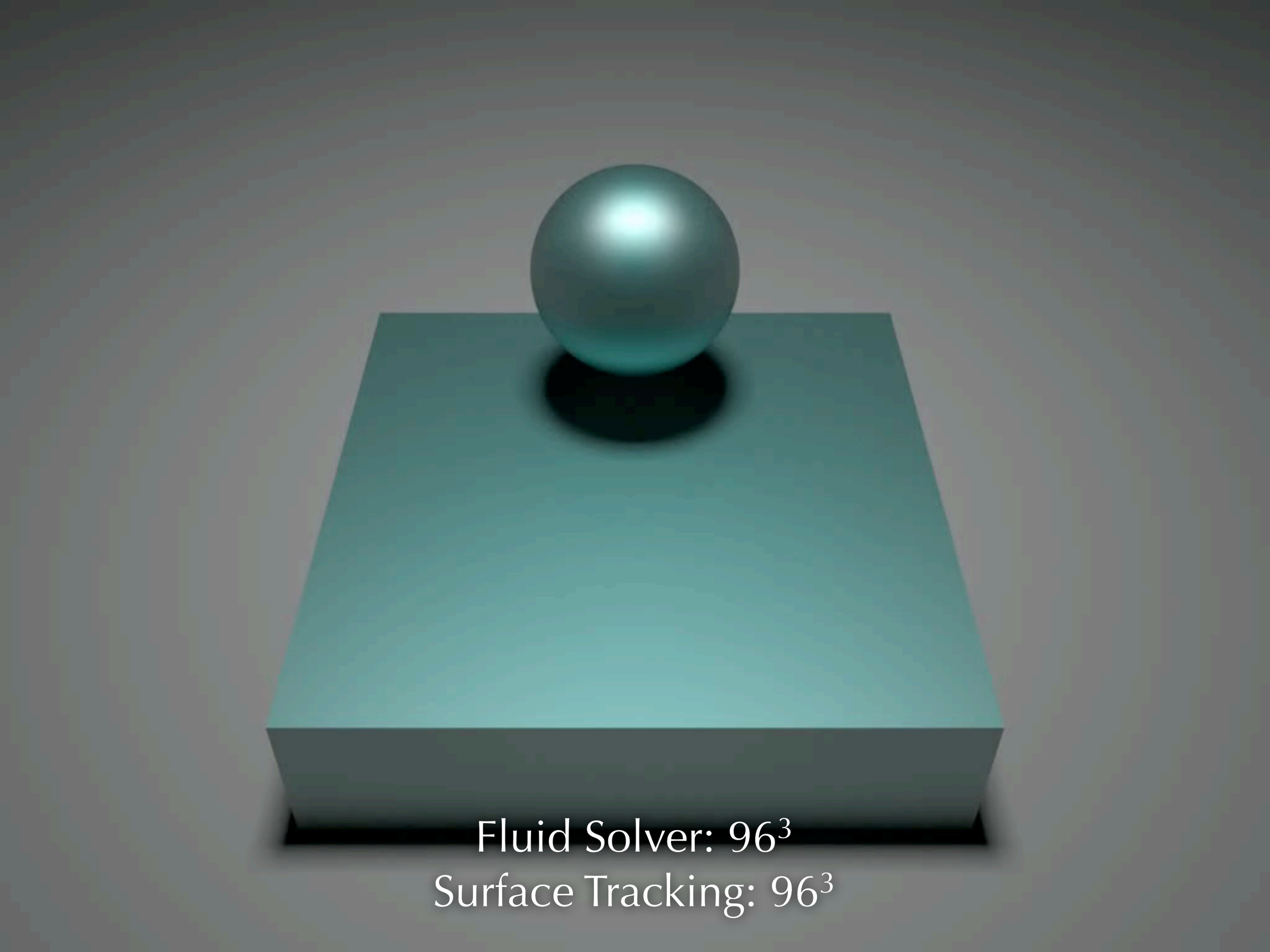
Enjoy!

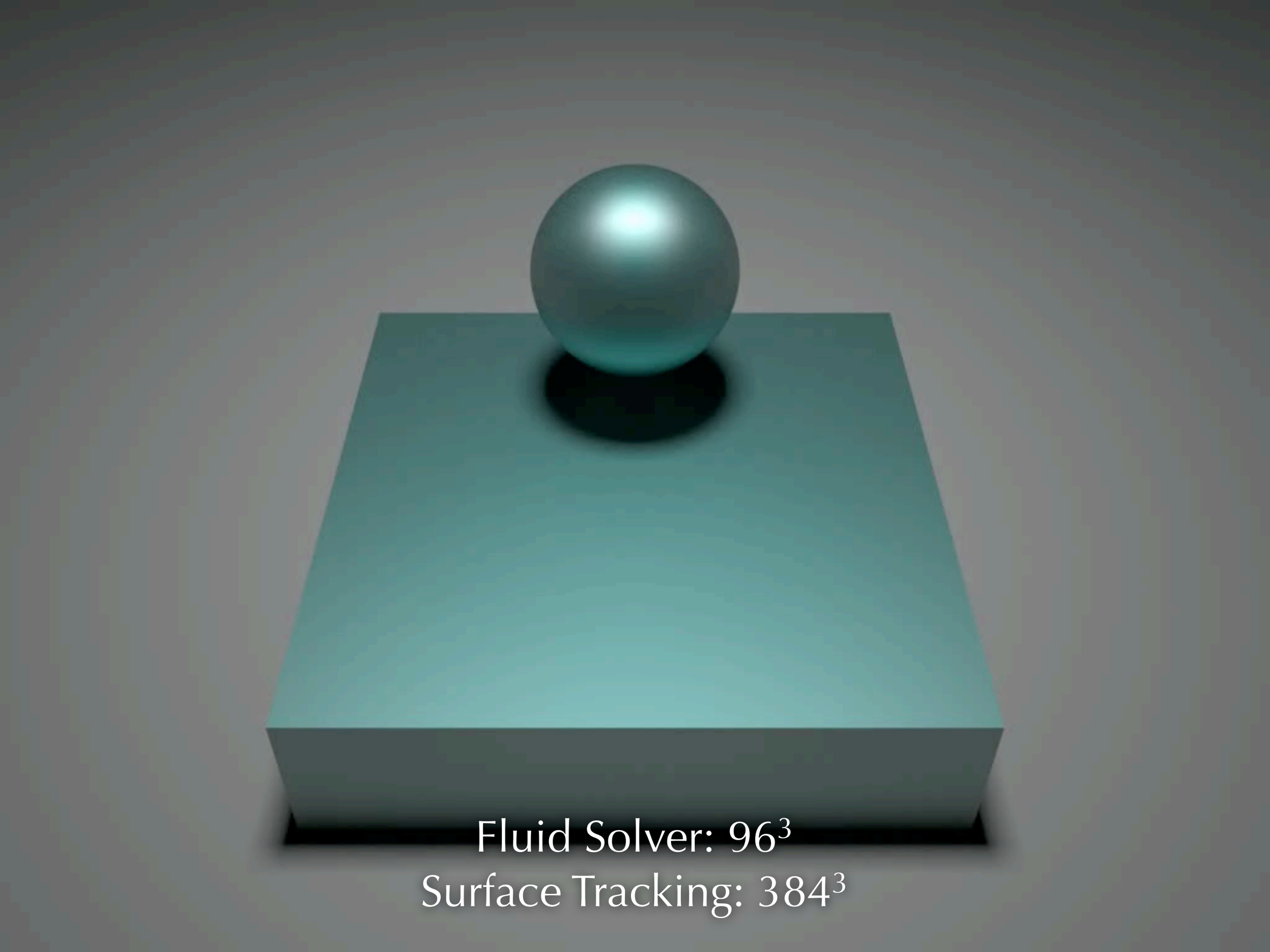



Fluid Solver: 96^3
Surface Tracking: 384^3


Fluid Solver: 96^3
Surface Tracking: 96^3

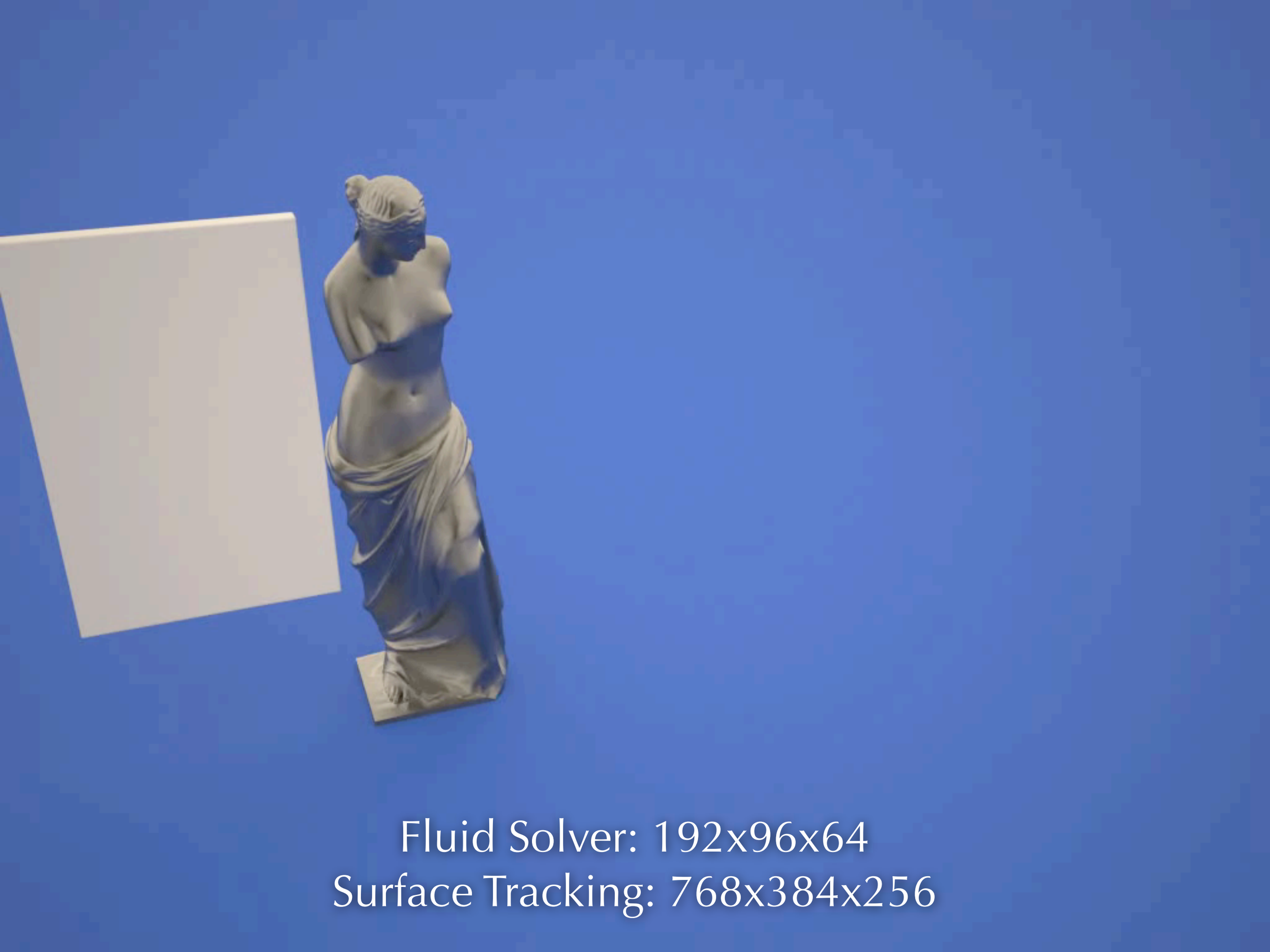

Fluid Solver: 128^3
Surface Tracking: 128^3


Fluid Solver: 96^3
Surface Tracking: 96^3


Fluid Solver: 96^3
Surface Tracking: 384^3

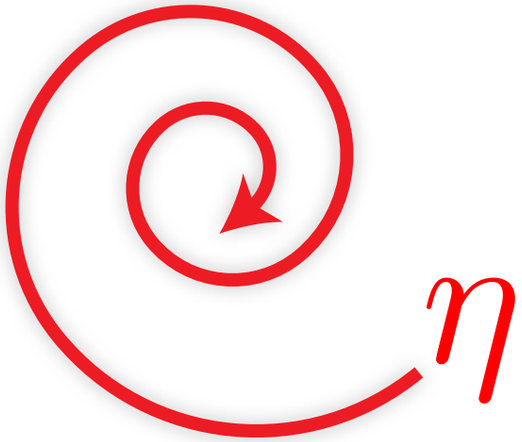

Fluid Solver: 160x80x80
Surface Tracking: 640x320x320

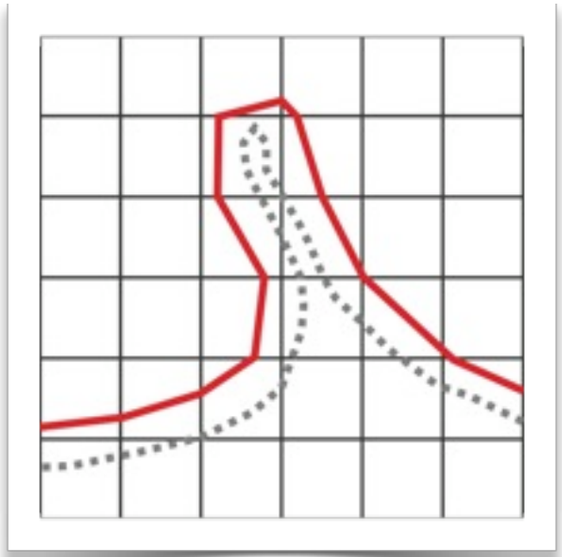

Fluid Solver: 192x96x64
Surface Tracking: 768x384x256

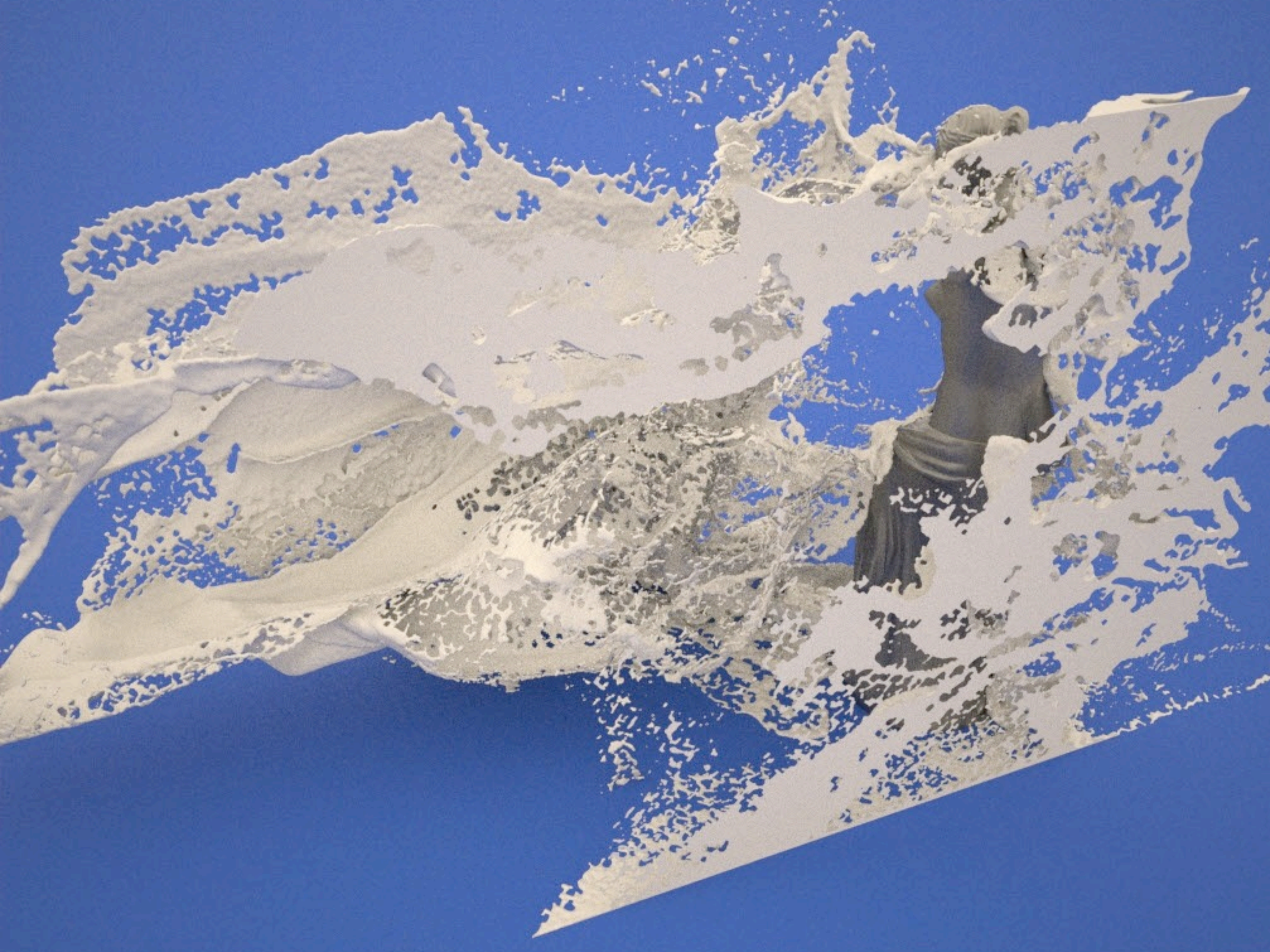

Fluid Solver: 192x96x64
Surface Tracking: 768x384x256

Conclusions

- Eulerian Vortex Sheet Method
  - Concentrated vortex at the phase interface
  - User parameter for turbulence-like effect
- Liquid-biased Filtering
  - Enables to properly down-sample high resolution mesh without aliasing
- Limitations
  - No consideration of bubbles
  - Still depends on resolution







Thank You

ありがとうございました

Any Questions?

Contact Info.
kim@graphics.snu.ac.kr